had a somewhat greater photographic intensity than the green line. For further details the Table in 'Roy Soc. Proc.,' A, vol. 101, p. 116, 1922, may be consulted.

No direct comparison of intensity for equal exposure at Terling and Shetland can be given. The uncertain weather in Shetland makes it an unfavourable locality for examining the general light of the night sky, apart from the visually distinguishable aurora which is usually only seen low down in the north.

The comparison I am able to give is between the latter and the general light of the sky at Terling, and it deals only with the intensity of the nitrogen bands relative to the green line. The northern lights in Shetland have a much greater absolute intensity, but it is seldom that a long exposure upon them can be secured.

Calculation of a Primary Standard of Mutual Inductance of the Campbell Type and Comparison of it with the Similar N.P.L. Standard.

By D. W. DYE, B.Sc., A.C.G.I.

INTRODUCTORY.

The standard mutual inductance devised and designed by Mr. A. Campbell and constructed in 1907–8* at the National Physical Laboratory has been one of the foundations of our alternating current measurements since that date.

It will be sufficient here to note that the special feature in the design of the Campbell type of mutual inductance consists in a primary single-layer winding, so proportioned that the field due to it is practically zero over the region occupied by the secondary coil. By this means the dimensions of the secondary coil are rendered relatively unimportant, so that it may be an overwound many-layer winding, whereby a suitably large value of mutual inductance may be obtained.

* A. Campbell, 'Roy. Soc. Proc.,' A, vol. 79, p. 428 (1907).

A number of fundamental measurements have also been made with its help, including a determination of the ohm[*] and a comparison of self-inductance standards of the Physikalische Technische Reichsanstalt.[†]   These measurements give, through various links, an independent check on the accuracy of our standard.   From general considerations of the probable accuracy which might be assigned to all the measurements it appeared possible that the uncertainty of knowledge of the value of this standard might be as great as 1 part in 10,000, *i.e.*, considerably greater than the uncertainty which would be assigned to it from the calculations and length measurements.   These latter together might amount to 2 or 3 parts in 100,000.

When, therefore, a similar standard of mutual inductance for the Japanese Government, which was under construction by Mr. R. W. Paul, was approaching completion and had shown great uniformity of the spiral groove finally cut on the marble cylinder at the Laboratory, it was thought that highly accurate measurements should be made of the diameter and pitch of the primary winding and also of the mean diameter and cross-section of the secondary winding in order that the value of the mutual inductance of this standard might be calculated to at least as great accuracy as the original N.P.L. standard.   With this end in view the primary winding was done at the Laboratory in order that the necessary measurements of the diameter of the bare copper wire under the tension of winding, might be made.   The secondary winding was also carried out at the Laboratory in order that the mean diameter of the successive layers of it might be measured as accurately as possible.   The calculations were made to the highest accuracy and the full corrections made for departure from mathematical exactness of the actual coils as measured.

The construction of this new standard differs only in minor details from the Laboratory standard; it is not thought that these details can have any measurable effect on the actual value as calculated.

WINDING OF THE PRIMARY AND SECONDARY COILS AND GENERAL REMARKS REGARDING THE NEW MUTUAL INDUCTANCE.

The marble cylinder was sent to the Laboratory for measurement in October 1913, and as a result of the report on these measurements, it was decided to have the two spiral grooves recut and rewound at the laboratory.   This, owing to the war, was delayed until the early part of 1920.   It was then taken in hand by the Engineering Department who recut the spiral and

[*] A. Campbell, 'Roy. Soc. Proc.,' A, vol. 87 (1912).
[†] 'N.P.L. Report,' p. 44 (1914–15).

rewound the two coils forming the primary winding. The dimensions and numbers of turns of the two primary windings are as follows :—

Approximate diameter	30 cm.
Length of coil	7·5 „
Distance between inside turns of coils	15 „
Number of turns in each coil	75

Some time after winding the primary coils, the cylinder was handed over to the Metrology Department and measured according to the scheme detailed below in Section 3 of this paper.

The secondary winding, which is a ring-shaped coil of many layers, was carried out in the Electrical Measurements Department. It is necessary to know the mean diameter represented by the central filament of the cross-section of the secondary, and since this winding consists of twenty layers of silk covered wire with a layer of paper between each, it was thought that the diameters of the central layers might be reduced slightly by the compression of the outer layers. The diameters of each successive layer can only be accurately measured at the time of winding it, before it is subjected to the compression of the outer layers.

In order to get an idea of the possible change in diameter of the layers due to this cause, holes were cut in the cheeks of the ring-shaped marble bobbin at the ends of three diameters spaced 120° apart. These holes allowed the outer wires in each layer to be seen and to be measured by means of a microscope after the whole coil had been wound. The outer wires at each side of the tenth layer were measured through these holes at the time of winding it and again after the whole twenty layers had been wound. The difference between the mean of the six measurements before and after winding the outer ten layers was within the experimental accuracy of the measurements, being less than 0·01 mm.

It may be taken, therefore, that the mean diameter as deduced from the diameters of each layer at the time of winding it represents accurately the mean diameter of the whole winding.

The bottom of the groove in the marble ring was too small to allow the secondary winding to be wound directly on it of the correct mean diameter. A single layer of wire of suitable diameter was, therefore, first wound on and a single layer of paper put over it in order to bring the inside diameter of the secondary winding to such a value as to give the correct mean diameter of the whole coil as nearly as could be determined beforehand.

The finished winding consists of twenty layers of double silk covered wire ; each layer has twenty-four turns and there is a strip of paper about 0·08 mm.

thick between each layer of wire. The odd five turns are in a twenty-first layer wound close together near the cheek of the ring at which the ends come out.

The winding is in four sections having the following numbers of turns, 96, 144, 144, 96 + 5 turns, thus making 485 turns in all when in series.

Length Measurements of the Japanese Standard.

(1) *Primary Winding on Marble Cylinder.*

(a) *Description.*—The two windings are referred to as P and Q throughout this paper, in accordance with the diagram, fig. 1, which also indicates the system of numbering adopted for the turns. The top of the cylinder is that end at which the outgoing leads are attached. The overall dimensions of the marble cylinder are :—
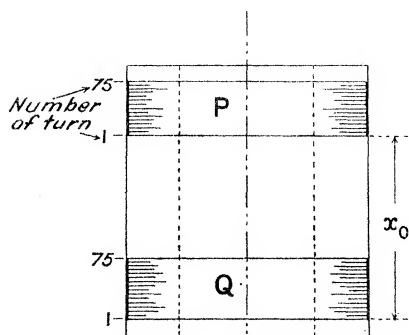

Number of turn

FIG. I.

Primary windings of mutual inductance.

Height .... 35 cm.
Outside diameter ...... 30 „
Inside diameter ...... 18 „

Each coil consists of seventy-five turns of bare wire, having a nominal pitch of 1 mm. and a nominal diameter of 30 cm. The distance x_0 is nominally 22·5 cm.

(b) *Measurement of Diameters.*—The diameters over the wires have been compared with the length of the Laboratory 300 mm. Hartmann Spherical-ended gauge. The variations in diameter are shown in fig. 2. The continuous portions of the curves, together with the extreme end points, are plotted from the calculated means of readings taken six at a time. The dotted lines completing the curves are partly estimated means.

It will be noted that the first turn of coil P has a diameter about 0·1 mm. less than that of the rest of the winding. This is due to the fact that the groove for this turn was added some time after that for the other turns, with the consequent error noted. The method used in the calculations for correction of variation of diameter takes quite accurate account of this comparatively large local variation in diameter.

The final mean diameters over the wires thus found were :—

Coil P = 300·778₇ mm. at 17° C. (omitting first turn)
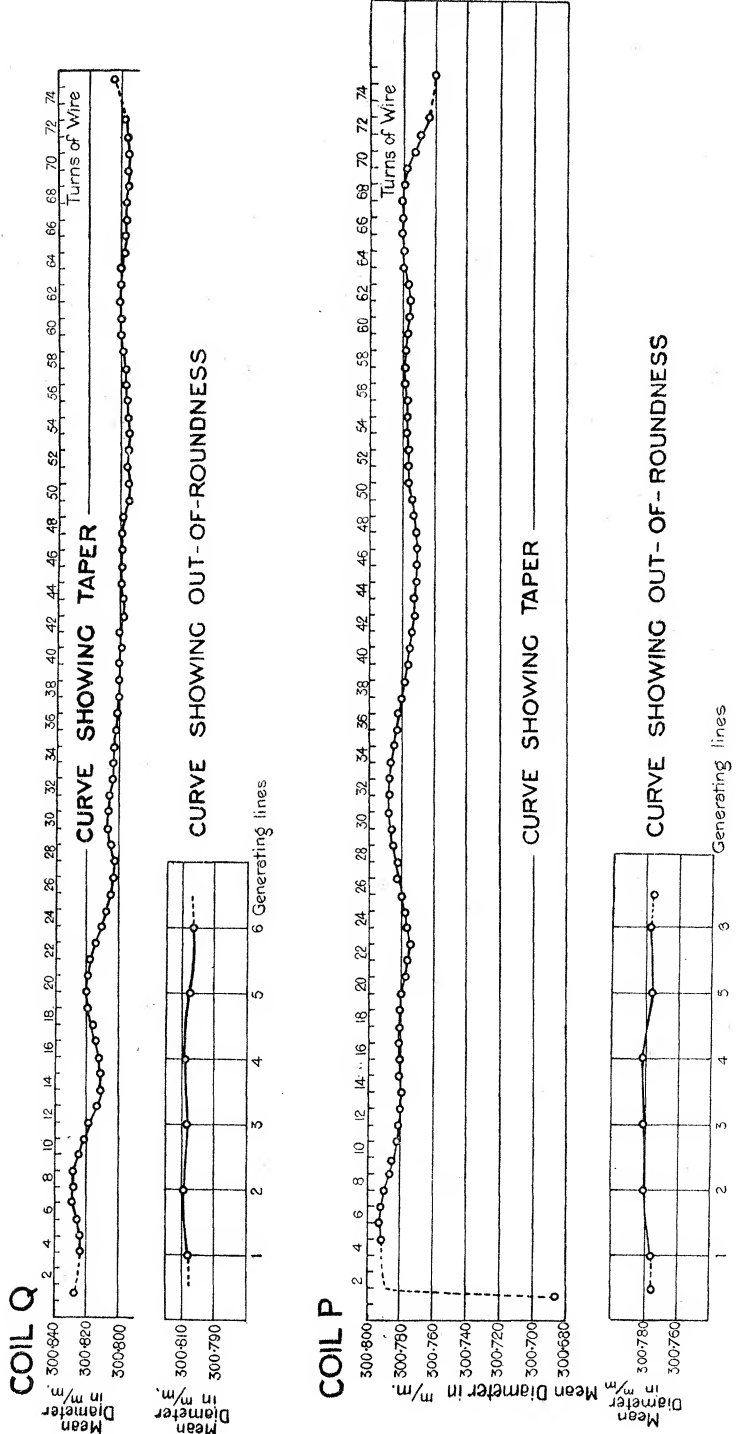Coil Q = 300·806₉ mm. at 17° C.

FIG. 2. MEAN DIAMETERS OF PRIMARY COILS OF STANDARD OF INDUCTANCE

In order to avoid fractional parts in the differences between the true diameter of each turn and the mean diameter, as given above, nominal mean diameters have been used having the values

$$\text{Coil P} = 300 \cdot 779 \text{ mm.}$$
$$\text{Coil Q} = 300 \cdot 807 \text{ mm.}$$

The differences between these nominal diameters and the actual diameter have been read off from the curves of fig. 2, and are given in Section (4), dealing with the corrections for variations from a perfect spiral.

(c) *Diameter of Wire.*—The diameter of the wire was measured at about 100 points for each coil, while it was under tension during the process of winding the wire on the cylinder. The mean diameters of wire thus found were

$$\text{Wire on Coil P} = 0 \cdot 620_8 \text{ mm.}$$
$$\text{Wire on Coil Q} = 0 \cdot 621_0 \text{ mm.}$$

(d) *Measurement of Pitch.*—The pitch has been measured against the Laboratory line standard Invar No. 27, whose calibration is accurately known. The nominal pitch is 1 mm., and the nominal mean distance from a point on coil P to the corresponding point on coil Q has been taken as $225 \cdot 445_8$ mm. All measurements on both coils are reckoned as from the first turn of coil Q, that is, readings on this turn are taken as correct. The nominal reading on any other turn minus the observed reading gives the variation from nominal pitch. These variations are shown in fig. 3. The continuous portions of the curves, together with the end points, have been plotted from the calculated means of readings taken six at a time, and the dotted portions completing the curves are partly estimated means.
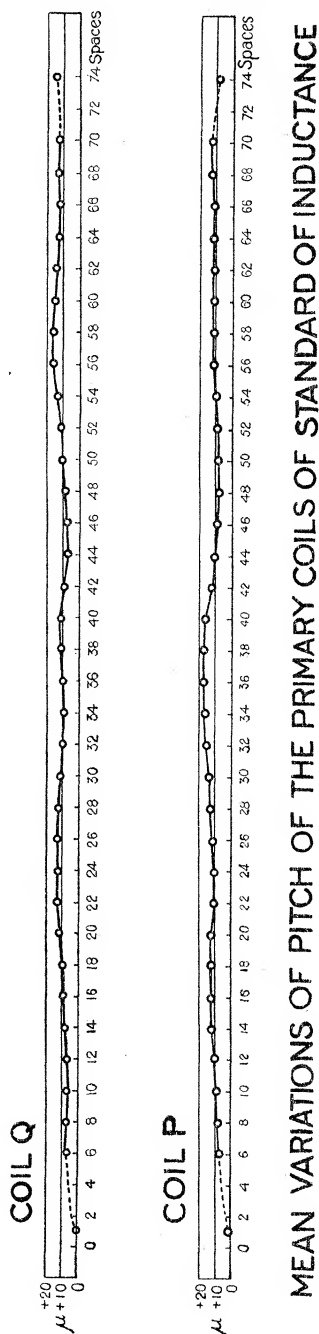

MEAN VARIATIONS OF PITCH OF THE PRIMARY COILS OF STANDARD OF INDUCTANCE

FIG. 3.

In order to correct for these variations in pitch, a zero line of ($+10\,\mu = 0.01$ mm.) has been chosen for each coil, and the axial displacements of every wire from the perfect spiral were taken from this base line : these, when multiplied by the appropriate dM/dx at that place, give the desired corrections. The change of the zero line for this purpose makes no difference in the fundamental calculations so long as the same change is made in both coils.

(e) *Irregularities at the Ends of the Windings.*—The ends of the windings come through small ivory plugs of 5 mm. diameter, which are let into recesses in the marble. The surfaces of these plugs are below the surface of the marble ; the wires consequently become chords instead of arcs of circles when passing over this space, and the end of the last turn also bulges outwards somewhat before the wire bends sharply in entering the hole in the plug. A calculation of the effects due to this departure from perfection of shape shows, however, that it is quite negligible. The change in mutual inductance due to a change in diameter of 0·2 mm. of one whole turn of the primary winding is of the order of 0·1 microh., *i.e.*, 1 part in 100,000 of the total mutual inductance.

There are also local irregularities of pitch near the ends of the windings : these also are of negligible effect.

(2) *Secondary Winding.*

The principle of the Campbell design of mutual inductance is such that the dimensions of the secondary winding are comparatively unimportant. They are, however, sufficiently important to warrant some care in the measurements of mean diameter and dimensions of cross-section of the winding.

The outside diameter of each layer was measured on diameters 120° apart by means of a large micrometer gauge, which was then immediately calibrated against a 17-inch standard spherical-ended gauge. The diameter of the silk-covered wire was also measured at a number of places during winding. The deduced values of mean diameter for each layer are given in Table I, p. 322.

The mean diameter for the whole coil is thus $437 \cdot 62_5$ mm. The mean diameter, as measured through the holes by microscope, was 437·63 mm. Since accuracy to 0·01 mm. in diameter is more than sufficient for an accuracy of 1 in 10^6 in the calculations, the mean diameter has been taken as 437·630 mm.

> The axial width of winding = Mean of six measurements through the
>   holes in the marble ring = 10·1 mm.
> Radial depth of winding = $9 \cdot 6_8$ mm.

Table I.—Mean Diameters of Successive Layers of Secondary Winding.

Layer.	Mean diameter.	Layer.	Mean diameter.	Layer.	Mean diameter.	Layer.	Mean diameter.
	mm.		mm.		mm.		mm.
1	$428 \cdot 35_8$	6	$433 \cdot 28_3$	11	$438 \cdot 10_0$	16	$442 \cdot 95_0$
2	$429 \cdot 34_5$	7	$434 \cdot 26_3$	12	$439 \cdot 08_0$	17	$443 \cdot 91_0$
3	$430 \cdot 33_7$	8	$435 \cdot 21_0$	13	$440 \cdot 02_0$	18	$444 \cdot 90_0$
4	$431 \cdot 31_0$	9	$436 \cdot 18_0$	14	$440 \cdot 99_0$	19	$445 \cdot 90_0$
5	$432 \cdot 28_9$	10	$437 \cdot 17_0$	15	$441 \cdot 98_0$	20	$446 \cdot 91_0$

(3) *Measurement of Thermal Coefficient of Expansion of the Marble.*

This was measured at temperatures ranging from $13°$ to $25°$, and found to be

$$+ 0 \cdot 000005_9 \text{ per } 1° \text{ C.}$$

(4) *Calculations and Corrections.*

The accurate formula of J. Viriamu Jones was used in the fundamental calculations. This formula gives the mutual inductance between a circle and a co-axial helix, having one end in the plane of the circle. A convenient form of this formula* is

$$\text{M} = 2\pi n_1 (\text{A} + a) \left\{ c/k \left(\text{F}(k) - \text{E}(k) \right) + \frac{\text{A} - a}{x} \psi \right\}, \qquad (1)$$

where   A = radius of circle, a = radius of helix, x = length of helix, n_1 = number of turns in the length "x" of the helix.

$$k^2 = \frac{4\text{A}a}{(\text{A} + a)^2 + x^2} = \sin^2 \gamma. \qquad c^2 = \frac{4\text{A}a}{(\text{A} + a)^2}.$$

$\text{F}(k)$ and $\text{E}(k)$ are complete elliptic integrals of the first and second kinds. They were computed by formulas (5) and (6),

$$\psi = \text{F}(k) \cdot \text{E}(k_1'\beta) - [\text{F}(k) - \text{E}(k)] \text{F}(k_1'\beta) - \frac{\pi}{2}$$

in which $\text{F}(k_1'\beta)$ and $\text{E}(k_1'\beta)$ are incomplete integrals of the second kind to modulus k' and amplitude β

$$k' = \cos \gamma \quad \text{and} \quad \beta = \sin^{-1} c'/k'; \qquad c' = \sqrt{(1 - c^2)}.$$

The values of mutual inductance obtained by the above formula have been multiplied by 485 in every case so as to refer them to a secondary winding of that number of turns.
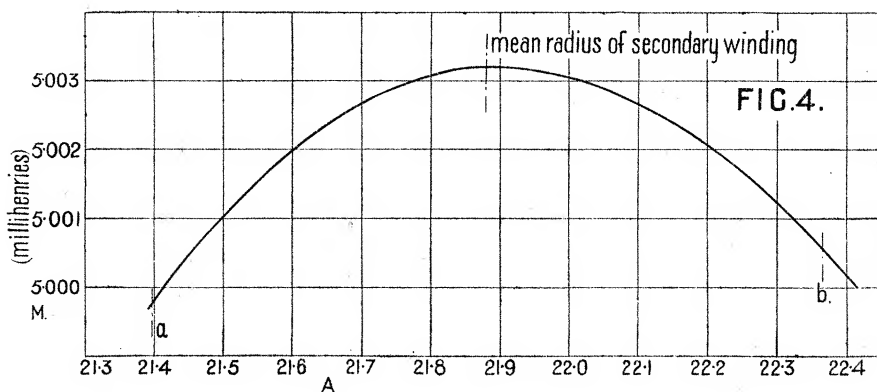
The calculations have been made in two parts, one for coil P and one for coil Q. The complete curve showing variation of mutual induction with

* A. Campbell, 'Roy. Soc. Proc.,' A, vol. 79, p. 434 (1907).

variation in diameter of the secondary winding, has only been made in the case of coil P. It was not essential to obtain the curve shown in fig. 4, but it forms a most valuable check on the calculation of M for the point required and also checks the curve shown in fig. 5 for dM/dA.

The dimensions used in the calculations for coil P were $a = 15\cdot00791$ cm., $x_1 = 15\cdot0223$ cm., $x_2 = 7\cdot5223$ cm.

Five values of A were taken, viz., $21\cdot3915$, $21\cdot6915$, $21\cdot8815$, $22\cdot0915$ and $22\cdot3915$. The reason these values of A were chosen was that it was thought

Variation of mutual inductance for variation of diameter of secondary circle.

that the maximum point would be at a radius of A equal to $21\cdot8915$ cm., this was afterwards found not to be the case.

The values obtained for the mutual inductances are given in Table II.

Table II.—Coil P and Secondary Circle of 485 Turns.

A (cm.).	M (x_1).	M (x_2).	M ($x_1 - x_2$).
21·3915	13·45049	8·45080	4·99969
21·6915	13·31803	8·31536	5·00267
21·8815	13·23494	8·23175	5·00319
22·0915	13·14390	8·14109	5·00281
22·3915	13·01502	8·01478	5·00024

The curve connecting A and M ($x_1 - x_2$) is given in fig. 4. For coil Q the values corresponding to its mean radius of $a = 15\cdot0093$ and the mean secondary radius A $= 21\cdot8815$ are,

A.	M ($x_1 = 15\cdot0223$).	M ($x_2 = 7\cdot5223$).	M ($x_1 - x_2 = 7\cdot5000$).
21·8815	13·23758	8·23354	5·00404

The sum of the mutuals to the circle secondary for coils P and Q is, therefore, $10\cdot00723$.

For averaging over the space occupied by the cross-section of the secondary winding the formula (2) has been used :—

$$M = M_0 + \frac{1}{a}\frac{dM}{dA}\left(\frac{c^2}{6} + \frac{20\,B^2C^2 - 9\,C^4}{360\,A_0^2}\right) + \left(\frac{B^2 - C^2}{6} + \frac{9\,C^4 - 20\,B^2C^2}{360\,A_0^2}\right)\frac{d^2M}{dA^2}$$

$$+ \frac{10\,B^2C^2 - 6\,C^4}{360\,A_0^2} \cdot \frac{d^3M}{dA^3} + \dots, \quad (2)$$

where $2\,B =$ radial depth and $2\,C =$ axial length of cross-section of the secondary.

In order to determine dM/dA, d^2M/dA^2, etc., the curve connecting dM/dA with A was found by calculation and plotting, using the equation
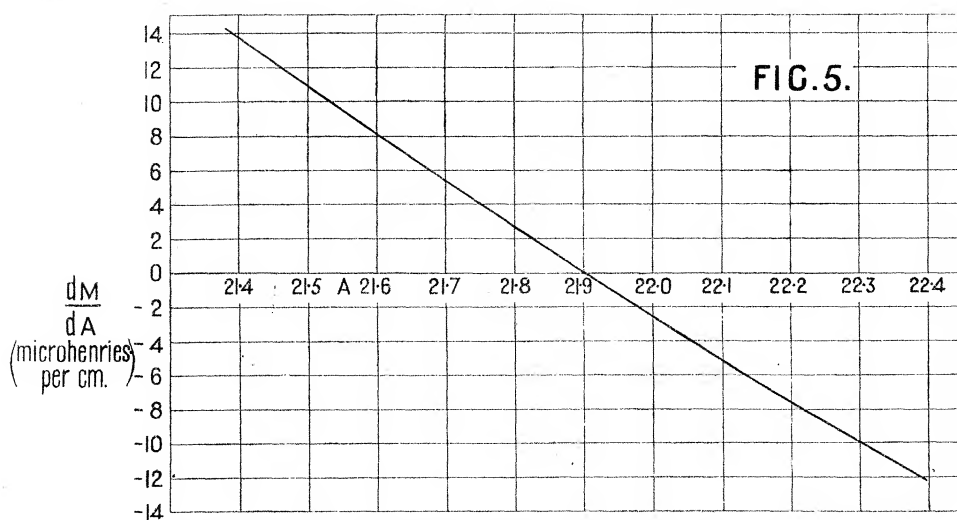
$$\frac{dM}{dA} = 2\pi\,n_1 n_2\,ck\left\{ F(k) + \left(\frac{A+a}{2ax}\,\sqrt{([A+a]^2 + x^2)}\right)\psi\right\} \quad (3)$$

in which the symbols have the same meaning as in equation (1) ; $n_2 = 485$.

The values obtained for dM/dA in the case of coil P are given in Table III.

Table III.

A.	dM/dA microhenries per centimetre.		
	$x_1 = 15\cdot0223.$	$x_2 = 7\cdot5223.$	$x_1 - x_2 = 7\cdot5000.$
21·3915	−444·11	−458·12	+ 14·01
21·6915	−438·96	−444·61	+ 5·65
21·8815	−435·67	−436·22	+ 0·55
22·0915	−432·00	−427·22	− 4·78
22·3915	−426·72	−414·74	− 11·98

Relation between dM/dA and A for coil P and secondary circle.

The curve connecting A and dM/dA for $(x_1 - x_2 = 7\cdot5000)$ is given in fig. 5. To a sufficient accuracy this curve is represented by the equation $dM/dA = -25\cdot8\,\epsilon + 4\cdot5\,\epsilon^2$ where ϵ is the difference between any value of A and that value corresponding to the maximum mutual inductance. In the present case A for the maximum mutual inductance, *i.e.*, when $dM/dA = 0$, is

$$A = 21\cdot90_1 \text{ cm. for Coil P, and}$$
$$A = 21\cdot90_4 \text{ cm. for Coil Q};$$

when ϵ is very small we have, therefore, for the whole mutual

$$\frac{dM}{dA} = -51\cdot6\epsilon, \qquad \frac{d^2M}{dA^2} = -51\cdot6, \qquad \frac{d^3M}{dA^3} = +18\cdot0.$$

In the present case for coil P, $\epsilon = -0\cdot0195$ cm., and $dM/dA = +0\cdot55$; for coil Q, $\epsilon = -0\cdot0225$ and $dM/dA = +0\cdot58$ microh. per centimetre.

The only term of any importance in equation (2) is the term

$$\left(\frac{B^2 - C^2}{6} + \frac{9\,C^4 - 20\,B^2 C^2}{360\,A_0^2}\right)\frac{d^2M}{dA^2}.$$

Substituting the values $B = 0\cdot48$ cm., and $C = 0\cdot50$ cm. (allowing for insulation on the wire) we get, for the correction on the total mutual inductance, due to the finite space occupied by the secondary, the result $+0\cdot170$ microh.

We have, therefore, for the case of the mutual inductance with two perfect spirals and a secondary winding of the dimensions given, the value.

$$M = 10\cdot00740 \text{ millih.}$$

Corrections for Axial and Radial Displacements of the Primary Winding.

Each turn of the primary helix is displaced axially and radially from its theoretical position. The most accurate method of determining the corrections due to these displacements is to calculate the dM/da and the dM/dx for each of a number of circles of radius "a" and different distances "x" from the larger circle A. By plotting these values against x, curves are obtained from which the dM/da and dM/dx for each turn of the primary winding may be read off. These values, when multiplied by the respective displacements radially and axially of each of the turns, give the small corrections on M due to them. The sum of all these corrections gives the total correction to be applied. The dM/da and dM/dx for the circles could have been calculated directly from the formulas for these quantities, but only one more calculation for each point is needed if the actual mutual inductances themselves are calculated; moreover, if the new elegant formulas developed from those for complete elliptic integrals are used for

the computation of the mutual inductances of the circles,* the labour involved is comparatively light, and no tedious and doubtful interpolation in the tables of elliptical integrals is required.

Starting off with the quantities a_0, b_0 and c_0 which have the values given below, Table IV is constructed.

Table IV.

a_0.	b_0.	c_0.
$a_1 = \frac{1}{2}(a_0 + b_0);$	$b_1 = \sqrt{(a_0 b_0)};$	$c_1 = \frac{1}{2}(a_0 - b_0).$
$a_2 = \frac{1}{2}(a_1 + b_1);$	$b_2 = \sqrt{(a_1 b_1)};$	$c_2 = \frac{1}{2}(a_1 - b_1).$
....	....	....
$a_n = \frac{1}{2}(a_{n-1} + b_{n-1});$	$b_n = \sqrt{(a_{n-1} \cdot b_{n-1})};$	$c_n = \frac{1}{2}(a_{n-1} - b_{n-1}).$

Then
$$M = \frac{2\pi^2}{a_n}(c_1{}^2 + 2c_2{}^2 + 4c_3{}^2 + \ldots) \qquad (4)$$

"a_0" and "b_0" converge with extreme rapidity to the same value. In general, for the cases under consideration, $a_3 = b_3$ to an accuracy of one part in ten million.

$$a_0 = \sqrt{(A+a)^2 + x^2}; \quad b_0 = \sqrt{(A-a)^2 + x^2}; \quad \text{and} \quad c_0 = \sqrt{(4Aa)}.$$

(*Note.*—c_0 is not required in this formula for mutual inductance.)

The expressions for $F(k)$ and $E(k)$ in terms of the above symbols are

$$F(k) = \frac{1}{2}\frac{\pi a_0}{a_n}, \qquad (5)$$

$$F(k) - E(k) = \frac{F(k)[c_0{}^2 + 2c_1{}^2 + 4c_2{}^2 + \ldots]}{a_0}. \qquad (6)$$

In computing the values of M for circles, three cases were taken for each value of "x" considered. They were

(1) A, a, x; (2) A, $a+0.01$ cm., x; (3) A, $x+0.01$ cm., a.

In these A $= 21.8815$ cm., $a = 15.00791$ cm., and $x = 7.500$ cm.
The difference (2) — (1) gives ΔM corresponding to $\Delta a = 0.01$ cm.
The difference (3) — (1) gives ΔM corresponding to $\Delta x = 0.01$ cm.

In carrying out these computations the constant terms $2\pi^2$ and $n_2 = 485$ have been left out of the calculations till the final additions of the corrections.

The values of M_1, ΔM_1 when $\Delta a = 0.01$ and ΔM_1 when $\Delta x = 0.01$ are given in Table V and curves figs. 6 and 7.

* L. V. King, "On some New Formulæ for the Numerical Calculation of the Mutual Inductance of Coaxial Circles," 'Roy. Soc. Proc.,' A, vol. 100, p. 60 (1921).
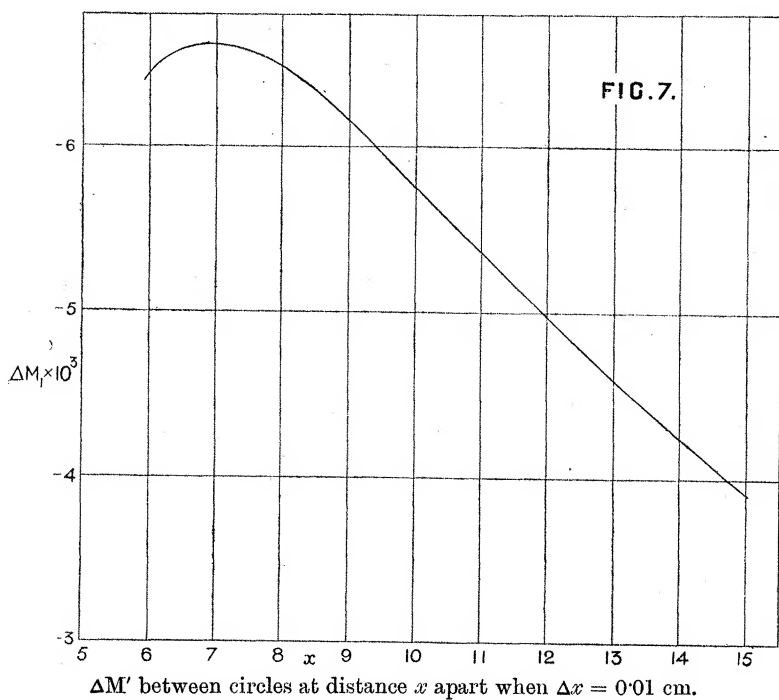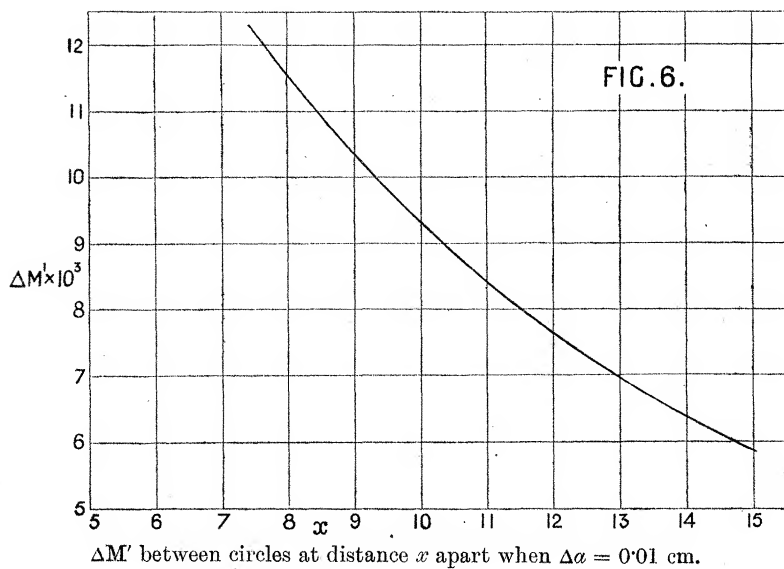
FIG.6.

$\Delta M' \times 10^3$

$\Delta M'$ between circles at distance x apart when $\Delta a = 0\cdot01$ cm.

FIG.7.

$\Delta M_1 \times 10^3$

$\Delta M'$ between circles at distance x apart when $\Delta x = 0\cdot01$ cm.

Table V. $M_1 = M/2\pi^2$ for Circles Radii $A = 21{\cdot}8815$ and $a = 15{\cdot}0079_1$
with various Values of x.

x.	M_1 (cm.).	ΔM_1 for $\Delta a = +0{\cdot}1$ mm.	ΔM_1 for $\Delta x = +0{\cdot}1$ mm.
6·0	10·14853	—	−0·00644
7·5	9·13485	+0·01217	−0·00658
8·0	8·80715	+0·01151	−0·00649
9·0	8·17523	+0·01032	−0·00615
10·0	7·57938	+0·00932	−0·00577
11·0	7·02249	+0·00843	−0·00537
12·0	6·50508	+0·00766	−0·00498
13·0	6·02630	+0·00699	−0·00459
14·0	5·58447	+0·00639	−0·00426
15·0	5·17742	+0·00588	−0·00390

The values of ΔM_1 corresponding to the position "x" of each turn have been read off from these curves and multiplied by the corresponding small differences u and v obtained from the curves, figs. 2 and 3. Since turn 1 on coil P and turn 75 on coil Q are the same distance "x" from the secondary winding, the algebraic sum of their radial and axial differences may be taken when multiplying by ΔM_1. A selection of the results of the calculations is given in Table VI, in which turn 1 now has the position "outer" on both coils and turn 75 the position "inner" on both coils.

Table VI.

Turn.	Sum of radial displacements on P and Q in units of 10^{-3} mm. $= u$.	Sum of axial displacements on P and Q in units of 10^{-3} mm. $= v$.	The unit is $1 = \dfrac{10^{-8} \text{ microh.}}{2\pi^2}$.	
			$u \times \dfrac{dM_1(a)}{dx}$.	$v \times \dfrac{dM_1(x)}{dx}$.
1	+ 1	+ 9	+ 5·9	− 35·0
5	+10	+ 7	+ 61·2	− 28·2
10	+17	+ 6	+ 108·3	− 25·2
15	+ 1	+ 4	+ 6·6	− 17·5
20	+12	0	+ 83·6	0
25	− 4	− 6	− 29·1	+ 28·5
30	− 7	− 3	− 53·3	+ 14·8
35	− 8	+ 6	− 63·9	− 30·7
40	− 3	+ 5	− 25·1	− 26·6
45	+ 2	+ 7	+ 17·6	− 38·4
50	− 8	+ 1	+ 74·2	− 5·7
55	−12	− 4	− 117·4	+ 23·7
60	− 4	− 4	− 41·2	+ 24·4
65	− 7	− 4	− 66·0	+ 25·1
70	+ 2	− 8	+ 22·9	+ 51·6
75	−97	−14	−1178·5	+ 92·0

The whole correction for the seventy-five turns of both windings of the primary was found to be—

For radial displacements of the wires $= -0·084$ microh.
For axial   ,,   ,, $= +0·015$,,

CORRECTION FOR THE FIVE TURNS FORMING THE TWENTY-FIRST LAYER.

The calculations relative to averaging over the cross-sectional area occupied by the secondary winding assume the whole of it to be contained within the rectangle formed by the twenty layers of twenty-four turns each; the extra five turns necessary to obtain the mutual inductance of 10 millih. are wound close together near the upper cheek of the ring. They may, to a sufficient accuracy, be considered as equivalent to a five-turn circle displaced radially by $+0·5$ cm. and axially by $0·4$ cm. from the central filament of the secondary winding.

The corrections due to these displacements are :—

For axial displacement $= +0·041$ microh.
For radial   ,, $= -0·061$,,
Net correction $= -0·020$,,

There is, finally, a correction due to one of the coils having slightly more than seventy-five turns; the overlap is $1·2$ mm. If the outgoing leads are close together, this extra piece may be considered as equivalent to a positive mutual inductance due to the part NP (Fig. 8) of the circle at one end of the helix together with a negative mutual given by the very open spiral QP formed by the outgoing lead from the end Q. QN is a generator of the cylindrical surface of the winding.
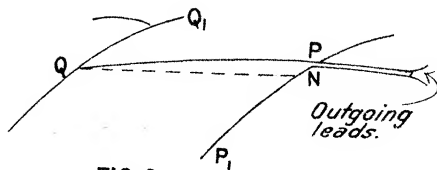
If the piece NP is considered as at the end of the helix closest to the secondary winding, the positive mutual inductance due to it will be greater than the negative mutual inductance due to the return lead QP; on the other hand, if the small piece is considered as at the end, Q, of the helix its positive value will be less than the negative value of QP. The length measurements of the distance separating the two helices are not quite sufficiently accurate to determine which case is correct.

FIG. 8.

Appertaining to overlap at one end of helix.

There is an uncertainty, also, due to the return piece QP being on a different cylinder to the helix, owing to the ends Q and P first going radially inwards through the walls of the hollow marble cylinder. The uncertainty introduced

by the ends before they come away together is probably of the order $\pm 3 \times 10^{-6}$.

Summing up the various calculations and corrections we have for the value of the standard,

For perfect Helix P .... $M = 5003 \cdot 19$ microh.

"   "   Q .... $M = 5004 \cdot 04$ "

	Microh.
Correction for averaging over cross-section of secondary	$+0 \cdot 170$
" for irregularities of diameter of primary helices ...	$-0 \cdot 084$
" " pitch of primary helices.........	$+0 \cdot 015$
" for five turns on outside of secondary winding......	$-0 \cdot 020$

These together give for the final calculated mutual inductance of the standard the value at 17° C.

$$M_{17} = 10 \cdot 0073_1 \text{ millih.}$$

Probable Accuracy of the Calculated Value of the Standard.

There are five factors entering into the calculations, each of which represents a series of length measurements. If a probable accuracy is given to these measurements it is possible to assign a probable accuracy to the final result. The factors involved are—

    (1) Diameter of primary helices.
    (2) Pitch and distance apart of primary helices.
    (3) Diameter of secondary winding.
    (4) and (5) Width and depth of secondary winding.

For the present purpose the helices may be replaced by a circle 10 cm. distant from the plane of the secondary winding.

If it is assumed that the diameter is accurate to $\pm 0 \cdot 001$ mm., and the pitch to $\pm 0 \cdot 002$ mm. for the primary winding; also that the mean diameter of the secondary winding is accurate to $\pm 0 \cdot 1$ mm., and the width and depth of the secondary cross-section is accurate to $\pm 0 \cdot 05$ mm., the following will be the resulting uncertainties in M expressed as parts in a million :—

For—Primary diameter..................... ± 6
       "   length and distance apart ± 16
Secondary diameter.................... $\pm 0 \cdot 7$
       "   depth and width ...... ± 8 each.

The maximum error is therefore 40 parts in 10^6, but the mean probable error will be of the order 20 parts in 10^6.

Comparison with the N.P.L. Standard Mutual Inductance.

(a) *Calculated Value of N.P.L. Standard.*—The original value calculated from measurements of dimensions made when the N.P.L. standard was first constructed in 1908, was

$$M_{15} = 10 \cdot 0178_5 \text{ millih. at } 15° \text{ C.}$$

In 1914, it was thought desirable to check the measurements of the standard. The paraffin-wax coating over the primary windings was dissolved off with petrol, and the pitch and diameter of these were again measured in the Metrology Department. It was found that changes in the dimensions had taken place, but whether these were secular, or due to the waxing or subsequent treatment with petrol, it is impossible to say. The diameters of the two helices were about 5 parts in 100,000 greater, and the axial lengths were all about 1 part in 10,000 greater. The net result of these changes was to make the value of the mutual inductance about 7 parts in 10^6 smaller. The value at 15° C., as determined from the second set of measurements, was

$$M_{15} = 10 \cdot 0177_8 \text{ millih.}$$

Owing to the uncertainties attaching to the effects of paraffin-waxing, it was decided not to wax either of the windings of the new standard. The windings can, therefore, be measured without trouble at any future time.

Comparison.

The two standards were compared by measuring each on a variable mutual inductometer. In order to avoid capacity effects a frequency of 10 cycles per second was used, at which frequency the sensitivity was about 3 parts in a million.

Observations were made alternately, first on one standard and then on the other. Balance on the new standard was, at first, found to be imperfect; this imperfection was traced and found to be due to metal parts in the levelling screws and sockets used to support and adjust the secondary winding. These were replaced by non-metallic parts, and, as a result, the balance was perfect, as far as could be judged on the galvanometer.

The results of four comparisons gave for the difference between the two standards the values

$$10 \cdot 6_2, \ 10 \cdot 7_0, \ 10 \cdot 6_3, \ 10 \cdot 6_0 \text{ microh.}$$

The mean difference at 16° C. may be taken, therefore, as

$$10 \cdot 6_4 \text{ microh.} \ (\pm \ 0 \cdot 05 \text{ microh.}).$$

The calculated value of the new standard at 17° C. is

$$M_{17} = 10\cdot0073_1 \text{ millih.}$$

If all the dimensions of a mutual inductance change in the same proportion the value of mutual inductance changes in that proportion also. The coefficient of thermal expansion of the marble of the new standard is $+0\cdot000006$ per 1° C.

The mutual inductance at any temperature " t " will therefore be

$$M_t = M_{17}\,[\,1 + 0\cdot00006\,(t-17)\,].$$

The N.P.L. standard has the value at 15° C.

$$\text{(N.P.L.)}\ M_{15} = 10\cdot0177_8 \text{ millih.}$$

Correcting the value of the new standard to this temperature we have

$$\text{(New)}\ M_{15} = 10\cdot0071_9 \text{ millih.}$$

The difference between the two calculated values is $10\cdot5_9$ microh. The mean measured difference, as given above, is $10\cdot6_4$ microh.

This agreement between calculation and measurement may be considered highly satisfactory; it is closer than the various probable errors might lead one to expect.

It appears, therefore, that both these standards are accurate to within about 1 part in 100,000, in their values calculated from the dimensions as measured in terms of the N.P.L. standards of length.